

Coherent population transfer in a chain of tunnel coupled quantum dots*

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We consider the dynamics of a single electron in a chain of tunnel coupled quantum dots, exploring the formal analogies of this system with some of the laser-driven multilevel atomic or molecular systems studied by Bruce W. Shore and collaborators over the last 30 years. In particular, we describe two regimes for achieving complete coherent transfer of population in such a multistate system. In the first regime, by carefully arranging the coupling strengths, the flow of population between the states of the system can be made periodic in time. In the second regime, by employing a “counterintuitive” sequence of couplings, the coherent population trapping eigenstate of the system can be rotated from the initial to the final desired state, which is an equivalent of the STIRAP technique for atoms or molecules. Our results may be useful in future quantum computation schemes.

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I. INTRODUCTION

Population transfer in multistate quantum systems has been an active topic of research over the last half a century. In the context of atomic and molecular physics, coherent population transfer in optically-driven multilevel systems has been studied since the invention of lasers [1]. Usually, the objective is to transfer the population from the initial to a well defined final state of the atom or molecule, via one or more intermediate states, while minimizing the loss of population through or its accumulation on the intermediate states. In early theoretical work, Shore and collaborators have studied population transfer in multilevel systems driven by resonant laser fields [2, 3, 4]. In particular, they have found that it is possible to arrange the coupling strengths between the adjacent states in such a way that the system becomes analogous to a spin- J in a magnetic field, whose dynamic evolution is known to be periodic for any J [3]. This coupling scheme was therefore named spin-coupling.

Later, Hioe, Eberly, Bergmann and collaborators discovered the technique of stimulated Raman adiabatic passage (STIRAP) for three-level atomic/molecular systems [5]. They have identified a specific eigenstate of the system, the so-called coherent population trapping (CPT) state, which contains a superposition of the initial and final states, and dates back to Alzetta et al. and Arimondo and Orriols [6]. The STIRAP technique is then based on first preparing the system in its initial bare state, which coincides with the CPT state, and then adiabatically rotating the CPT state towards the desired final bare state of the system. This technique has been subsequently polished [7] and extended to multilevel systems [8, 9, 10] with the active participation of Bruce W. Shore.

While the above studies were conducted in the context of multilevel atoms or molecules, here we show

that similar effects can be found in the context of quantum transport in arrays of tunnel-coupled quantum dots [11, 12, 13, 14]. Often referred to as artificial atoms, semiconductor quantum dots offer an unprecedented possibility of constructing at will and exploring situations ranging from practically single atom to a fully solid state many-body systems [15]. The nanofabrication possibilities of tailoring structures to desired geometries and specifications, and controlling the number and mobility of electrons confined within a region of space, are some of the features that make these structures unique tools for the study of a variety of preselected set of phenomena, including the coherent population transfer in multistate systems.

Given the controllable quantum properties of the electrons in such structures, the possibility of their application to schemes of quantum computers (QCs) [16] has not escaped attention [17, 18, 19]. The qubits of the QD-array based QC would be represented by the spin-states of single electrons confined in individual QDs, with the two-qubit nearest-neighbor coupling mediated by the controlled spin-exchange interaction [17, 18]. One of the main difficulties with the existing proposals for integrated solid-state based QCs is that there is no efficient way of transferring the information between distant qubits. We consider here a single-electron tunneling in a one-dimensional array of QDs and establish the conditions under which the complete transfer of the electron wavepacket between two distant locations can be achieved. Our findings could therefore be relevant to the reliable information exchange between distant parts of an integrated quantum computer [20].

In Section II we outline the mathematical formalism describing a chain of QDs, in terms of which, in Section III, we present the theory of coherent propagation and periodic oscillations of the electron wavepacket between the two ends of the chain. The single-electron transfer via an equivalent of multistate STIRAP is discussed in Section IV. In Section V we describe an envisioned implementation of a scalable quantum computer, followed by the concluding remarks.

*This paper is dedicated to Bruce W. Shore on the occasion of his 70th birthday.

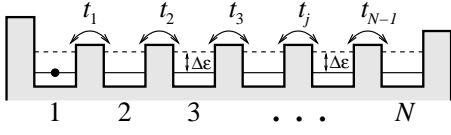


FIG. 1: Schematic drawing of the chain of tunnel-coupled QDs.

II. MATHEMATICAL FORMALISM

We consider electron transport in a linear array of N nearly identical QDs which are electrostatically defined in a two-dimensional electron gas by means of metallic gates on top of a semiconductor heterostructure (GaAs/AlGaAs) [11, 15]. This system is described by the extended Mott-Hubbard Hamiltonian [12, 13], which in its most general form is given by

$$H = \sum_{j,\alpha} \varepsilon_{j\alpha} a_{j\alpha}^\dagger a_{j\alpha} + \frac{1}{2} \sum_j U n_j (n_j - 1) + \sum_{i<j,\alpha} t_{ij,\alpha} (a_{i\alpha}^\dagger a_{j\alpha} + a_{i\alpha} a_{j\alpha}^\dagger) + \sum_{i<j} V_{ij} n_i n_j, \quad (1)$$

where $a_{j\alpha}^\dagger$ and $a_{j\alpha}$ are the creation and annihilation operators for an electron in state α with the single-particle energy $\varepsilon_{j\alpha}$, U is the on-site Coulomb repulsion, $n_j = \sum_\alpha a_{j\alpha}^\dagger a_{j\alpha}$ the total electron number operator of the j th dot, $t_{ij,\alpha}$ are the coherent tunnel matrix elements between dots i and j , and V_{ij} is the interdot electrostatic interaction. In general, the index α refers to both orbital and spin states of an electron. In the tight-binding regime, when the on-site Coulomb repulsion and single-particle level-spacing $\Delta\varepsilon$ are much larger than the tunneling rates, $U > \Delta\varepsilon \gg t_{ij,\alpha}$, only the equivalent states of the neighboring dots are tunnel-coupled to each other [21]. In the absence of a magnetic field, we can thus limit our consideration only to a single doubly- (spin-) degenerate level per dot ($\alpha \in \{\uparrow, \downarrow\}$), assuming further that the tunneling rates do not depend on the electron spin.

In this paper we are concerned with single-electron dynamics, considering a situation in which a preselected QD is initially doped with one mobile electron, while all of the other dots of the chain are empty, as indicated in Fig. 1. Our aim is to determine the conditions under which the complete coherent transfer of the electron between the two ends of the chain can be achieved. The population transfer in this system is mediated by the tunneling between the neighboring QDs. The individual tunneling rates $t_j \equiv t_{j,j+1}$ are determined by the voltages applied to the gates defining the corresponding interdot tunneling barriers. A chain of N tunnel-coupled QDs doped with a single electron is described by the following Hamiltonian,

$$H_{1e} = \sum_{j,\alpha} \varepsilon_{j\alpha} a_{j\alpha}^\dagger a_{j\alpha} + \sum_{j,\alpha} t_j (a_{j\alpha}^\dagger a_{j+1,\alpha} + a_{j\alpha} a_{j+1,\alpha}^\dagger), \quad (2)$$

which obviously does not contain terms responsible for electrostatic interactions. Since this Hamiltonian pre-

serves the electron number and its spin, the total state-vector of the system reads

$$|\psi(\tau)\rangle = \sum_{j,\alpha} A_j^\alpha(\tau) |j_\alpha\rangle, \quad (3)$$

where $|j_\alpha\rangle \equiv a_{j\alpha}^\dagger |0_1, \dots, 0_N\rangle$ denotes the state with one electron having spin α at the j th dot. The time-evolution of the system is governed by the Schrödinger equation $i\dot{|\psi\rangle} = H_{1e} |\psi\rangle$ ($\hbar = 1$), which yields

$$i \frac{dA_j^\alpha}{d\tau} = \varepsilon_j A_j^\alpha + t_{j-1} A_{j-1}^\alpha + t_j A_{j+1}^\alpha, \quad (4)$$

where $t_0 = t_N = 0$. Obviously, the two sets of these amplitude equations with $\alpha = \uparrow$ and $\alpha = \downarrow$ are equivalent and decoupled from each other. As a result, if the electron is prepared in an arbitrary superposition of spin up and spin down states, $|\psi\rangle = A_j^\uparrow |j_\uparrow\rangle + A_j^\downarrow |j_\downarrow\rangle$, the two parts of the wavefunction evolve symmetrically and independently of each other. This assertion is valid as long as all the uncontrollable spin-flip processes are vanishingly small on the time scale of t^{-1} . In semiconductor QDs, the spin decoherence originates mainly from the spin-phonon coupling, as well as the coupling of the electron spin with the nuclear spins of the surrounding crystal (hyperfine interaction) or stray magnetic fields. The first decoherence mechanism is suppressed at low temperatures [21], at which the density of crystal phonons is negligible [22]. As for the uncontrollable hyperfine interactions, experimental measurements indicate spin-relaxation times in excess of 100 μ s, which can be further improved by applying moderate magnetic fields or polarizing the nuclear spins [23]. Another mechanism for decoherence in the process of electron (charge) transfer in our system originates from the structure imperfections and gate voltage fluctuations, which cause uncertainty in the intradot energy levels and interdot couplings. These fluctuations, however, are typically slow on the time scale of t^{-1} , and the resulting disorder in the system may be considered frozen during its dynamic evolution, as we have discussed in a previous publication [20].

Let us write the Hamiltonian for the electron with spin α in the matrix form

$$H_{1e}^\alpha = \begin{bmatrix} \varepsilon_1 & t_1 & 0 & \cdots & & \\ t_1 & \varepsilon_2 & t_2 & & & \\ 0 & t_2 & \varepsilon_3 & & & \\ \vdots & & & \ddots & & \vdots \\ & & & & \varepsilon_{N-1} & t_{N-1} \\ & & & \cdots & t_{N-1} & \varepsilon_N \end{bmatrix}, \quad (5)$$

which is obviously tridiagonal. Inspection of the amplitude equations (4) or the Hamiltonian (5) indeed verifies that our system is formally analogous to the laser-driven multilevel atomic or molecular systems studied by Shore and coworkers [2, 3, 4] and Bergmann, Shore and others [5, 7, 8, 9, 10]. Here, the tunneling rates t_j between states

$|j\rangle$ and $|j+1\rangle$ play the same role as the Rabi frequencies of the laser fields acting on the atomic transitions $|j\rangle \leftrightarrow |j+1\rangle$, while the energies ε_j of states $|j\rangle$ correspond to the cumulative detunings of the atomic levels. In the following Sections, we describe two methods for achieving complete population transfer from the initial $|1\rangle$ to the final $|N\rangle$ state of the system, which turn out to be the counterpart of those in Refs. [3] and [10].

III. PERIODIC OSCILLATIONS OF POPULATION BETWEEN THE TWO END STATES

In this Section we consider the electron wavepacket dynamics in the chain with static couplings between the dots. Assume that at time $\tau = 0$ the electron is localized on the first dot, $|\psi^\alpha(0)\rangle = |1_\alpha\rangle$, and the tunnel couplings are switched on. This switching should be fast enough on the time scale of t^{-1} , so that no appreciable change in the initial state of the system occurs during the switching time τ_{sw} , but slow on the time scale of ε^{-1} , so that no nonresonant coupling between the dots is induced: $\varepsilon^{-1} < \tau_{\text{sw}} < t^{-1}$. The aim is to determine the set of couplings between the states of the systems which will achieve a complete transfer of the electron population from the initial to the final dot.

To determine the time-evolution of the state vector (3) we need to solve the eigenvalue problem $H_{1e}^\alpha |\psi^\alpha\rangle = \lambda |\psi^\alpha\rangle$ which will yield the eigenvalues λ_k and corresponding eigenvectors $|\psi_k^\alpha\rangle$ of the Hamiltonian (5). The state vector $|\psi^\alpha(\tau)\rangle$ at any time $\tau \geq 0$ is given by

$$|\psi^\alpha(\tau)\rangle = \sum_k^N e^{-i\lambda_k\tau} |\psi_k^\alpha\rangle \langle\psi_k^\alpha|\psi^\alpha(0)\rangle = \sum_j^N A_j^\alpha(\tau) |j_\alpha\rangle. \quad (6)$$

Note that the matrix in Eq. (5) has the form of the tridiagonal Jacobi matrix. It is natural to first consider the case of equal tunneling rates between the dots: $t_j = t$. Assuming equal energies $\varepsilon_j = \varepsilon$ and making the transformation $A_j^\alpha \rightarrow A_j^\alpha e^{i\varepsilon\tau}$, which is equivalent to the interaction picture, we find that the determinant $\mathcal{D}_N(\lambda) \equiv \det(H_{1e}^\alpha - \lambda\mathbb{I})$ is identical to the Chebyshev polynomial of the second kind, which can be expressed as $\mathcal{D}_N(\lambda) = \Pi_{k=1}^N (\lambda - \lambda_k)$. The eigenenergies of the system are then given by the roots of this polynomial, namely

$$\lambda_k = 2t \cos\left(\frac{k\pi}{N+1}\right),$$

while the corresponding eigenvectors are

$$|\psi_k^\alpha\rangle = \sqrt{\frac{2}{N+1}} \sum_j^N \sin\left(\frac{jk\pi}{N+1}\right) |j_\alpha\rangle.$$

Using Eq. (6) and the initial conditions $A_1 = 1$ and $A_j = 0$ for $j = 2, 3, \dots, N$, we obtain the solutions for the

amplitudes as,

$$A_j^\alpha = \frac{2}{N+1} \sum_{k=1}^N \exp\left[-i2t\tau \cos\left(\frac{k\pi}{N+1}\right)\right] \times \sin\left(\frac{jk\pi}{N+1}\right) \sin\left(\frac{k\pi}{N+1}\right). \quad (7)$$

It is thus evident that the eigenstates of the coupled system oscillate with incommensurate frequencies corresponding to the roots λ_k of \mathcal{D}_N , which in fact become increasingly densely spaced with increasing N . As a consequence, the system never revives fully to its initial state, as is illustrated in Fig. 2(a).

Clearly, it is highly desirable to tailor the parameters of the system so as to achieve a non-dispersive transfer of the single-electron wavepacket between the two ends of the chain. Recall from the theory of angular momentum that a spin- J particle subject to a constant magnetic field exhibits Larmor precession about the field direction. In particular, if one chooses the quantization direction along an axis perpendicular to the magnetic field direction and prepares the particle in its lowest spin eigenstate $|J, M = -J\rangle$, it will oscillate between this initial and the final state $|J, M = J\rangle$ in a perfectly periodic way. The matrix elements for the transitions $|J, M\rangle \leftrightarrow |J, M+1\rangle$ between the neighboring states are proportional to $\sqrt{(J-M)(J+M+1)}$. It is therefore clear that with the appropriate choice of the interdot tunneling matrix elements, the dynamics of the single-electron in a chain of QDs can mimic that of a spin- J in a magnetic field. Indeed, if we formally set $N = 2J + 1$ and $j = J + M + 1$, the tunneling rates t_j should be arranged according to $t_j = t\sqrt{(N-j)j}$ for $j = 1, \dots, N-1$. Then again, by exploring the properties of the Jacobi polynomials, we find equally spaced eigenenergies of the system,

$$\lambda_k = t(2k - N - 1),$$

while the corresponding eigenvectors can be expressed through the rotation matrices commonly used in the representation theory of angular momentum. With the initial conditions $A_1 = 1$ and $A_j = 0$ for $j = 2, 3, \dots, N$, for the amplitudes of the state-vector (3), we then obtain simple analytic expressions given by the binomial form

$$A_j^\alpha = \left(\frac{N-1}{j-1}\right)^{1/2} [-i \sin(t\tau)]^{(j-1)} \cos(t\tau)^{(N-j)}. \quad (8)$$

Since the eigenstates of the system have commensurate energies λ_k , the electron wavepacket oscillates in a perfectly periodic way between the first and the last dots, whose occupation probabilities are given, respectively, by $|A_1^\alpha|^2 = \cos(t\tau)^{2(N-1)}$ and $|A_N^\alpha|^2 = \sin(t\tau)^{2(N-1)}$, which is illustrated in Fig. 2(b). In particular, if at time $\tau = \pi/(2t)$ the tunneling rates are suddenly switched off, we obtain $|A_1^\alpha|^2 = 0$ and $|A_N^\alpha|^2 = 1$, i.e. complete population transfer from the initial to the final state of the

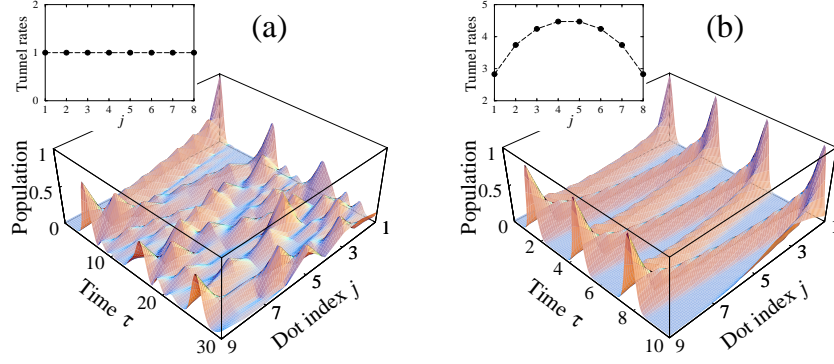


FIG. 2: Time-evolution of a single-electron wavepacket in a chain of $N = 9$ QDs with static tunneling rates. (a) Population flow in the chain with equal interdot tunneling rates $t_j = t$ (shown in the inset). (b) Population flow in the chain with spin-model tunneling rates $t_j = t\sqrt{(N-j)j}$ (shown in the inset). The time τ is in units of t^{-1} .

system. In a somewhat abstract sense, the behavior of the system is thus similar to that of a two-level system subject to a π pulse. Let us note at this point that the population transfer between the two ends of the chain can be achieved most straightforwardly by sequentially pulsing the tunneling rates between the first and second dots for time $\tau_1 = \pi/(2t_1)$, then the second and third dots for time $\tau_2 = \pi/(2t_2)$, etc till reaching the N th dot, which is equivalent to applying a sequence of π pulses in a multistate atomic system. In the scheme described above, however, all the interdot tunnelings are switched on and then off simultaneously, realizing thereby a fast and efficient transfer of the electron from the first to the last QD.

IV. ADIABATIC POPULATION TRANSFER BETWEEN THE TWO END STATES

While the above tunneling schemes, involving a sequence of π pulses or an effective collective π pulse, require both, careful control of the individual tunneling rates and their timing, in this Section we describe a robust adiabatic method for population transfer which is not very sensitive to small uncertainties in the interdot tunneling rates. Recall that a three-level atom interacting with two laser fields, under the condition of two-photon (Raman) resonance, possesses a coherent population trapping (CPT) state, which is decoupled from both laser fields [7]. Equivalently, for a chain of three tunnel-coupled quantum dots, assuming equal energies $\varepsilon_j = \varepsilon$, the eigenstate of Hamiltonian (5) with zero eigenvalue, $\lambda_0 = 0$, is given by

$$|\psi_0^\alpha\rangle = \frac{1}{\sqrt{\mathcal{N}_0}}[t_2 |1_\alpha\rangle - t_1 |3_\alpha\rangle], \quad \mathcal{N}_0 = t_1^2 + t_2^2. \quad (9)$$

This is a CPT state that does not contain a contribution from the intermediate state $|2_\alpha\rangle$. The other two

eigenstates

$$|\psi_\pm^\alpha\rangle = \frac{1}{\sqrt{\mathcal{N}_\pm}}[t_1 |1_\alpha\rangle - \lambda_\pm |2_\alpha\rangle + t_2 |3_\alpha\rangle],$$

$$\mathcal{N}_\pm = t_1^2 + \lambda_\pm^2 + t_2^2 = 2\mathcal{N}_0,$$

with corresponding eigenvalues $\lambda_\pm = \pm\sqrt{t_1^2 + t_2^2}$, contain all three states $|j_\alpha\rangle$. If for a given coupling strengths t_1 and t_2 the system is prepared in the CPT state (9), it will remain in this state as long as the couplings are constant in time. But even for time-dependent couplings, the system initially prepared in the CPT state can adiabatically follow this state, provided the tunneling rates change slowly enough. More quantitatively, the nonadiabatic coupling between the eigenstates of Hamiltonian (5) is small, if during the evolution the transition amplitude $\langle\psi_\pm^\alpha|\dot{\psi}_0^\alpha\rangle$ remains much smaller than the energy separation between the corresponding eigenstates [7],

$$|\langle\psi_\pm^\alpha|\dot{\psi}_0^\alpha\rangle| \ll |\lambda_\pm - \lambda_0|. \quad (10)$$

Our objective is to transfer the electron from the first to the last QD using the time-dependent (pulsed) tunnel-couplings. From Eq. (9) one can see that if at an early time the tunnel coupling t_2 is switched on while $t_1 \ll t_2$, the CPT state coincides with the initial state $|1_\alpha\rangle$. One then slowly (adiabatically) decreases t_2 while increasing t_1 , so that at a later time $t_1 \gg t_2$ and the CPT state coincides with the final state $|3_\alpha\rangle$. Assuming that t_2 and t_1 are represented by partially overlapping pulses, each having temporal width τ_w , the adiabaticity condition (10) requires $t_{1,2}^{\max}\tau_w \gg 1$.

In the field of atomic/molecular physics, this technique, involving the so-called counterintuitive sequence of pulses, is known as the stimulated Raman adiabatic passage (STIRAP) that is commonly used for coherent population transfer in three-state systems [7]. We note that the solid-state implementations of the CPT and STIRAP in a pair of coupled quantum dots driven by two electromagnetic fields has been proposed in [24]. The single electron transfer in a chain of three QDs via counterintuitive

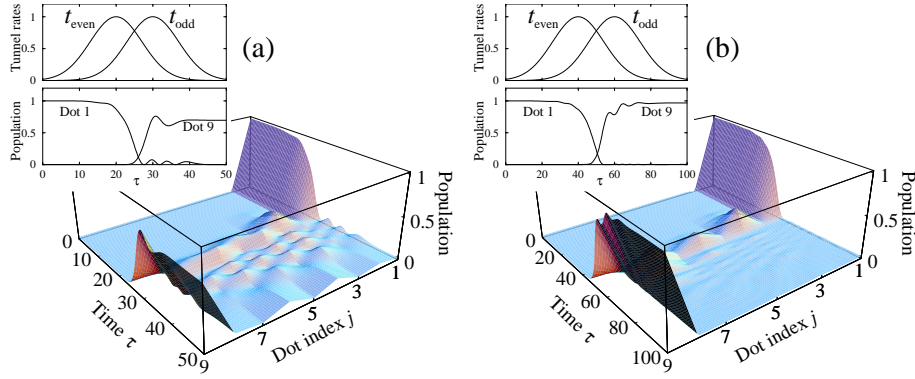


FIG. 3: Time-evolution of a single-electron wavepacket in a chain of $N = 9$ QDs with time-dependent, counterintuitive tunneling rates. (a) Population transfer is incomplete, $|A_N|^2 \simeq 0.7$, when the adiabatic condition is not very well satisfied. (b) Almost complete population transfer is achieved, $|A_N|^2 \simeq 0.97$, when the adiabatic condition is better satisfied by doubling the temporal widths of the pulses and the total interaction time (note the different scales of the time axis in (a) and (b)). The insets show the time-dependence of even and odd tunneling rates and the populations of the first and last QDs.

pulsing of tunnel-couplings as discussed above has been studied by Greentree et al. in [14], where it was termed coherent tunneling by adiabatic passage (CTAP). These authors also considered the extension of CTAP to multidot systems employing the so-called straddling scheme of [9]. Other schemes for adiabatic electron transport in tunnel-coupled QDs have been discussed in [25].

Another extension of the STIRAP technique to systems containing more than just three states has been given in [10]. This scheme can easily be adapted to our system, as described below. We thus consider a chain of N sequentially coupled QDs and assume that the individual tunnel couplings can selectively and independently be manipulated. When N is odd, i.e. $N = 3, 5, 7, \dots$, the Hamiltonian (5) has a CPT eigenstate

$$|\psi_0^\alpha\rangle = \frac{1}{\sqrt{\mathcal{N}_0}} [t_2 t_4 \dots t_{N-1} |1_\alpha\rangle + (-1)^J t_1 t_4 \dots t_{N-1} |3_\alpha\rangle + \dots + (-1)^J t_1 t_3 \dots t_{N-2} |N_\alpha\rangle], \quad (11)$$

$$J \equiv \frac{1}{2}(N-1),$$

with eigenvalue $\lambda_0 = 0$. Thus the amplitude of the initial state $|1_\alpha\rangle$ is proportional to the product of all the even-numbered tunnel-couplings, while the amplitude of state $|N_\alpha\rangle$ is given by the product of all odd-numbered tunnel-couplings, divided by the normalization parameter $\mathcal{N}_0 = (t_2 t_4 \dots t_{N-1})^2 + \dots + (t_1 t_3 \dots t_{N-2})^2$. Therefore, if all the even-numbered tunnel-couplings are pulsed together first, the CPT state (11) would coincide with the initial state $|1_\alpha\rangle$. This is then followed by switching-on all the odd-numbered tunnel-couplings, while the even-numbered ones decrease, which will result in a complete transfer of electron wavepacket to the state $|N_\alpha\rangle$. If we assume that these two families of pulses are described by common shape functions, $t_2, t_4, \dots, t_{N-1} = t_{\text{even}}$ and

$t_1, t_3, \dots, t_{N-2} = t_{\text{odd}}$, Eq. (11) takes a compact form

$$|\psi_0^\alpha\rangle = \frac{1}{\sqrt{\mathcal{N}_0}} \sum_{n=0}^J (-t_{\text{odd}})^n t_{\text{even}}^{J-n} |(2n+1)_\alpha\rangle, \quad (12)$$

$$\mathcal{N}_0 = \sum_{n=0}^J t_{\text{odd}}^{2n} t_{\text{even}}^{2(J-n)},$$

which makes the above discussion more transparent. In particular, complete population transfer from the initial state $|1_\alpha\rangle$ to the final state $|N_\alpha\rangle$ can be achieved by applying first the t_{even} pulses and then the t_{odd} pulses, the two sets of pulses partially overlapping in time, as shown in Fig. 3.

In order to minimize the nonadiabatic coupling of the CPT state to other eigenstates of the system, the rate of change of t_{even} and t_{odd} , given approximately by the inverse pulse-width τ_w^{-1} , should be small compared to corresponding eigenenergies $|\lambda| \sim |t_{\text{even}} + t_{\text{odd}}|$, which yields the same condition as above, $t_{\text{even,odd}}^{\text{max}} \tau_w \gg 1$. One can see from the results in Fig. 3(a), which were obtained precisely for this reason, that when this condition is not very well satisfied, the population transfer is incomplete. As expected, when the tunneling rates are pulsed for longer times, or, equivalently, have larger amplitudes, the adiabaticity condition is satisfied better, resulting in the complete population transfer from the initial to the final dot of the chain, as seen in Fig. 3(b). The remarkable advantage of this method over the one described in the previous Section is that as long as the two sets of partially overlapping pulses are strong enough, the adiabatic transfer of population is expected to be robust with respect to small uncertainties and fluctuations of tunneling rates, just like its atomic/molecular counterpart in Refs. [5, 7, 10]. On the other hand, the electron transfer via effective collective π pulse can be achieved with smaller tunneling rates and/or reduced interaction times, provided a precise control of the tunneling amplitudes and timings is

possible. Depending on the characteristics of the particular system, one or the other method may prove to be more practical.

V. CONCLUSIONS

In the above Sections, we have studied the dynamics of a single-electron transport in a linear array of tunnel coupled quantum dots. We have identified two regimes under which a complete coherent transfer of electron wavepacket between the two ends of the array can be achieved. Our results could be used for reliable information exchange between distant parts of an integrated quantum computer. As already noted in the Introduction, one of the difficulties with the existing proposals for integrated QD based QCs [17, 18] is that the qubits (electron spins) interact with the nearest neighbors only, and there is no efficient way of transferring the information between distant qubits. As a way around such difficulties, one can envision an integrated quantum register composed of a large number of sub-registers, each containing two or more adjacent qubits, represented by spins of single electrons in individual QDs. The sub-registers are embedded in a two-dimensional array of empty QDs. As we have shown in an earlier publication [20], through the mechanism of transient Heisenberg coupling, combined with the control of tunnel-coupling between the dots studied in this paper, this two-dimensional grid could realize a flexible quantum channel, capable of

connecting any pair of qubits within the register. Thus, to transfer the information, one connects distant sub-registers by a chain of empty QDs and applies one of the protocols described in the previous Sections to achieve a non-dispersive transfer of the qubit, followed by its controlled entanglement with a target qubit [17]. Note that this scheme is analogous to a proposal for an integrated ion trap based QC [26], where, in order to circumvent the difficulties associated with a single large ion trap quantum register, it has been proposed to use many small sub-registers, each containing only a few ions, and connect these sub-registers to each other via controlled qubit (ion) transfer to the interaction region (entangler) represented by yet another ion trap.

We should note that the coherent electron dynamics in arrays of tunnel-coupled QDs bears many analogies with spin-wave dynamics in spin chains [27] or electromagnetic field dynamics in periodic photonic crystals [28, 29], where some of the effects described above should be observable. With an unprecedented control over system parameters, arrays of QDs doped with more than one electron allow for studies of numerous coherence and correlation effects in many-body physics.

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